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F-term Inflation in M-theory with Five-branes

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Abstract

We study F-term inflation in M-theory with and without five-brane moduli fields. We show the slow rolling condition is not satisfied in M-theory without five-brane moduli fields, but it can be satisfied in the case with non-vanishing F-terms of five-brane moduli fields.

1 Introduction

Cosmological inflation can solve the flatness and horizon problems of the universe [1]. During inflation, the vacuum energy $V_0 \equiv < V >$ takes a value $V_0 = 3H^2M^2$, where M is the reduced Planck scale and H is expected to be of $O(10^{13} \sim 10^{14})$ GeV. The successful inflation scenario also requires the so-called slow-rolling condition for the inflaton field Φ, that is, the parameters $\varepsilon_{in} = \frac{1}{2}M^2(V'/V)^2$ and $\eta = M^2V''/V$ should be suppressed,

$$\varepsilon_{in} << 1, \qquad \eta << 1. \tag{1}$$

Within the framework of supergravity, the scalar potential V includes the F-term and D-term contributions,

$$V = F^I \bar{F}^{\bar{J}} \partial_I \partial_{\bar{J}} K - 3e^G M^4 + (D\mathrm{-term}). \tag{2}$$

Here K denotes the Kähler potential and G is obtained as $G = KM^{-2} + \ln(|W|^2M^{-6})$, where W is the superpotential. Thus, the nonvanishing vacuum energy V_0 can be induced by nonvanishing F-terms and/or D-terms. That implies supersymmetry breaking.

Here we consider the case that F-terms contributions are dominant within the framework of string-inspired supergravity [2, 3, 4], although the D-term inflation [5] induced by the anomalous $U(1)$ symmetry [6] is also another interesting possibility. Within the framework of superstring theory, the dilaton and moduli fields can be candidates for the inflation field [2]. However, here we consider other matter fields as candidates for the inflaton in hybrid inflation, which is driven by the vacuum energy due to non-vanishing F-terms of the dilaton and moduli fields.

In general, non-vanishing F-terms, however, induce a seizable soft scalar mass m_Φ for the inflaton field Φ, and that spoils the flatness condition (1) of V for Φ. This problem has been discussed within the framework of weakly coupled superstring theory. In Ref.[7] it has been shown that we have $m_\Phi^2 = 0$ for the field with the modular weight $n = -3$ and the vacuum energy V_0 driven by the F-term of the moduli field T in weakly coupled heterotic string theory without nonperturbative Kähler potential.

In this paper we consider the condition $m_\Phi^2 = 0$ for $V_0 \neq 0$ within the framework of strongly coupled heterotic sting theory, M-theory [8]. We take into account effects due to five-brane moduli fields.

2 M-theory without five-brane

First we consider the case without five-brane. The Kähler potential K is obtained [9],

$$K = -\log(S+\bar{S}) - 3\log(T+\bar{T}) + \left(\frac{3}{T+\bar{T}} + \frac{\alpha}{S+\bar{S}}\right)|\Phi|^2, \quad (3)$$

where $\alpha = 1/(8\pi^2)\int \omega \wedge \left[tr(F\wedge F) - \frac{1}{2}tr(R\wedge R)\right]$. The fields S and T denote the dilation field and moduli field. We assume that the F-terms of S and T contribute to V_0. Then we parameterize the F-terms,

$$F^S = \sqrt{3}m_{3/2}C(S+\bar{S})\sin\theta, \quad (4)$$

$$F^T = m_{3/2}C(T+\bar{T})\cos\theta, \quad (5)$$

where $C^2 = 1 + V_0/(3m_{3/2}^2 M^2)$ and $m_{3/2} = e^G M^2$. We have neglected the CP phases of F-terms. Using this parametrizatioin, the soft scalar mass m_Φ^2 is obtained [10],

$$\begin{aligned} m_\Phi^2 &= V_0 M^{-2} + m_{3/2}^2 - \frac{3C^2 m_{3/2}^2}{3(S+\bar{S})+\alpha(T+\bar{T})} \\ &\times \left\{\alpha(T+\bar{T})\left(2 - \frac{\alpha(T+\bar{T})}{3(S+\bar{S})+\alpha(T+\bar{T})}\right)\sin^2\theta \right. \\ &+(S+\bar{S})\left(2 - \frac{3(S+\bar{S})}{3(S+\bar{S})+\alpha(T+\bar{T})}\right)\cos^2\theta \\ &\left. -\frac{2\sqrt{3}\alpha(T+\bar{T})(S+\bar{S})}{3(S+\bar{S})+\alpha(T+\bar{T})}\sin\theta\cos\theta\right\}. \end{aligned} \quad (6)$$

We take the limit $C^2 >> 1$, i.e. $V_0 >> 3m_{3/2}^2 M^2$, and investigate the condition for $m_\Phi^2 << O(H^2 = V_0 M^{-2})$. It is obvious that we have $m_\Phi^2 = O(H^2)$ in most of the parameter space. However, the condition $m_\Phi^2 = 0$ is realized for the following case:

$$\cos\theta = 0, \qquad \frac{(S+\bar{S})}{\alpha(T+\bar{T})} = 0. \quad (7)$$

Unfortunately, the solution (7) is not a realistic solution from the viewpoint of M-theory. In M-theory, we have two sectors, which are usually called

the observable sector and the hidden sector. The sector including the scalar field Φ has the gauge kinetic function f [11],

$$f = S + \alpha T, \tag{8}$$

and the other sector has the gauge kinetic function f',

$$f' = S + \alpha' T. \tag{9}$$

The gauge couplings of these sectors g and g' are obtained as $g^{-2} = Re(f)$ and $g'^{-2} = Re(f')$. The coefficients α and α' should satisfy

$$\alpha + \alpha' = 0. \tag{10}$$

It is impossible that the both of g^{-2} and g'^{-2} take positive values for the condition (7), i.e. $(S+\bar{S}) << \alpha(T+\bar{T})$. Thus, the condition $m_\Phi^2 << O(H^2)$ can not be realized in the F-term inflation of M-theory without five-brane.

3 M-theory with five-branes

Next we consider M-theory with five-brane moduli fields Z^n [12], whose vacuum expectation values provide with positions z^n of the corresponding five-branes along the orbifold S^1/Z_2. The moduli Kähler potential K_{mod} and the Kähler potential of the scalar field Φ K_Φ are obtained

$$\begin{aligned} K_{mod} &= -ln(S+\bar{S}) - 3ln(T+\bar{T}) + K_5, \qquad (11)\\ K_\Phi &= \left(\frac{3}{(T+\bar{T})} + \frac{\epsilon\zeta}{(S+\bar{S})}\right)|\Phi|^2, \qquad (12) \end{aligned}$$

where

$$\zeta = \beta + \sum_{n=1}^{N}(1-z^n)^2\beta^{(n)}. \tag{13}$$

Here K_5 denotes the Kähler potential for Z^n, ϵ is the expansion parameter $\epsilon = (\kappa/4\pi)^{2/3}\, 2\pi^2\rho/V^{2/3}$, $\beta^{(n)}$ is a magnetic charge on the each 5-brane and β is the instanton number on the sector boundary including the inflaton Φ. The gauge kinetic functions of the sector including the inflaton Φ and the

other sector, f and f', are obtained in the same forms as eqs.(8) and (9) except changing α and α',

$$\begin{aligned}
\alpha &= \epsilon\left(\beta + \sum_{n=1}^{N}(1-z^n)^2\beta^{(n)}\right), \\
\alpha' &= \epsilon\left(\beta' + \sum_{n=1}^{N}(z^n)^2\beta^{(n)}\right).
\end{aligned} \tag{14}$$

We have the constraint

$$\beta + \beta' + \sum_{n=1}^{N}\beta^{(n)} = 0. \tag{15}$$

The condition (10) is relaxed by $\beta^{(n)}$. Thus the solution (7) could be realistic and both g^{-2} and g'^{-2} could be positive if both α and α' are positive, or if one of them is positive and the other is suppressed enough, e.g. $\alpha > 0$ and $\alpha' \approx 0$.

Furthermore, the F-term of the five-brane moduli Z^n can also contribute to V_0 and m_Φ^2. That changes the situation and could give a realistic solution even for $\alpha\alpha' < 0$. In this section, we consider effects of the F-term of Z^n.

For simplicity, we assume that there is only one relevant 5-brane moduli Z and its Kähler potential is a function of only $(Z + \bar{Z})$. Now we can parameterize each F-term as follows

$$\begin{aligned}
F^S &= \sqrt{3}m_{3/2}C(S+\bar{S})\sin\theta\sin\phi, \\
F^T &= m_{3/2}C(T+\bar{T})\cos\theta\sin\phi, \\
F^Z &= \sqrt{\frac{3}{K_{5,Z\bar{Z}}}}m_{3/2}C\cos\phi.
\end{aligned} \tag{16}$$

In addition, we fix z and $\beta^{(1)}$, e.g.

$$z = \frac{1}{2}, \qquad \beta^{(1)} = \frac{4}{3}\beta. \tag{17}$$

Note that in this case we have

$$2Re(f) = (S+\bar{S}) + \epsilon\zeta(T+\bar{T}), \tag{18}$$

$$2Re(f') = (S+\bar{S}) - \frac{3}{2}\epsilon\zeta(T+\bar{T}), \tag{19}$$

i.e. $\alpha\alpha' < 0$, where $\alpha = \epsilon\zeta$ and $\alpha' = -3\epsilon\zeta/2$.

In the limit $C >> 1$, we obtain m_Φ^2 [13] *,

$$
\begin{aligned}
m_\Phi^2 \;=\;& 3m_{3/2}^2 C^2 \Bigg\{ 1 - \Big[1 - \frac{9(S+\bar{S})^2}{\Big(3(S+\bar{S}) + \epsilon\zeta(T+\bar{T})\Big)^2}\Big] \sin^2\theta \sin^2\phi \\
&- \frac{1}{3}\Big[1 - \Big(\frac{\epsilon\zeta(T+\bar{T})}{(3(S+\bar{S}) + \epsilon\zeta(T+\bar{T})}\Big)^2\Big] \cos^2\theta \sin^2\phi \\
&- \frac{\epsilon\zeta(T+\bar{T})}{K_{5,Z\bar{Z}}\Big(3(S+\bar{S}) + \epsilon\zeta(T+\bar{T})\Big)} \Big[2 - \frac{\epsilon\zeta(T+\bar{T})}{(3(S+\bar{S}) + \epsilon\zeta(T+\bar{T})}\Big] \cos^2\phi \\
&+ \frac{2\sqrt{3}(S+\bar{S})\epsilon\zeta(T+\bar{T})}{\Big(3(S+\bar{S}) + \epsilon\zeta(T+\bar{T})\Big)^2} \sin\theta\cos\theta\sin^2\phi \\
&- \frac{2\sqrt{3}(S+\bar{S})\epsilon\zeta(T+\bar{T})}{\Big(3(S+\bar{S}) + \epsilon\zeta(T+\bar{T})\Big)^2} \sqrt{\frac{3}{K_{5,Z\bar{Z}}}} \sin\theta\sin\phi\cos\phi \\
&+ \frac{2\epsilon\zeta(T+\bar{T})}{3(S+\bar{S}) + \epsilon\zeta(T+\bar{T})} \Big[1 - \frac{\epsilon\zeta(T+\bar{T})}{(3(S+\bar{S}) + \epsilon\zeta(T+\bar{T})}\Big] \\
&\times \sqrt{\frac{1}{3K_{5,Z\bar{Z}}}} \cos\theta\sin\phi\cos\phi \Bigg\}. \qquad (20)
\end{aligned}
$$

The region with $\cos\theta = 0$ in eq.(6) gives a solution for $m_\Phi^2 = 0$ without five-brane, although it is not realistic because of negative gauge couplings squared. Hence, let us consider the condition $m_\Phi^2 = 0$ for $\cos\theta = 0$ in eq.(20) at first. For $K_{5,Z\bar{Z}} = 1$, the equation becomes simple and the two solutions for $m_\Phi^2 = 0$ are obtained

$$
\frac{(S+\bar{S})}{\alpha(T+\bar{T})} = 0, \qquad \frac{(S+\bar{S})}{\alpha(T+\bar{T})} = \frac{2}{3}\sin\phi\cos\phi. \qquad (21)
$$

While the former corresponds to the solution (7) in the case without five-brane, the latter is a new solution. However, the latter is not realistic, either, because the value $\alpha(T+\bar{T})$ is still large compared with $(S+\bar{S})$ and it can not lead to positive g^{-2} and g'^{-2} at the same time.

*See also Ref.[14].

Next, let us consider the $K_{5,Z\bar{Z}}$ dependence varying it for $\cos\theta = 0$. We fix ϕ, e.g. $\tan\phi = 1$. On top of that, the gauge coupling constant of the observable sector g_{GUT} must satisfy $2g_{GUT}^{-2} \simeq 4$. Thus, if the field Φ belongs to the observable sector, we take

$$(S + \bar{S}) + \alpha(T + \bar{T}) = 4. \tag{22}$$

On the other hand, if the field Φ belongs to the hidden sector, we take

$$(S + \bar{S}) - \frac{3}{2}\alpha(T + \bar{T}) = 4. \tag{23}$$

In the former case (22) the positivity of g^{-2} and g'^{-2} requires $\alpha(T+\bar{T}) < 8/5$, while in the latter case (23) $\alpha(T + \bar{T}) > -8/5$ is required. Fig.1 shows the solution of $m_\Phi^2 = 0$ leading to such values of $\tau = \alpha(T + \bar{T})$. The lower and upper lines correspond to the cases (22) and (23), respectively. In the case that Φ belongs to the observable sector, $1/K_{5,Z\bar{Z}} > 3.4$ is required for $\alpha(T + \bar{T}) < 8/5$. In the case that Φ belongs to the hidden sector, the same region $1/K_{5,Z\bar{Z}} > 3.4$ leads to positive values of $\alpha(T + \bar{T})$, that is, $\alpha(T + \bar{T}) > -8/5$ is satisfied. In this region, we have the realistic solutions of $m_\Phi^2 = 0$ for $\cos\theta = 0$ and $\tan\phi = 1$ when the field Φ belongs to the observable or hidden sector.

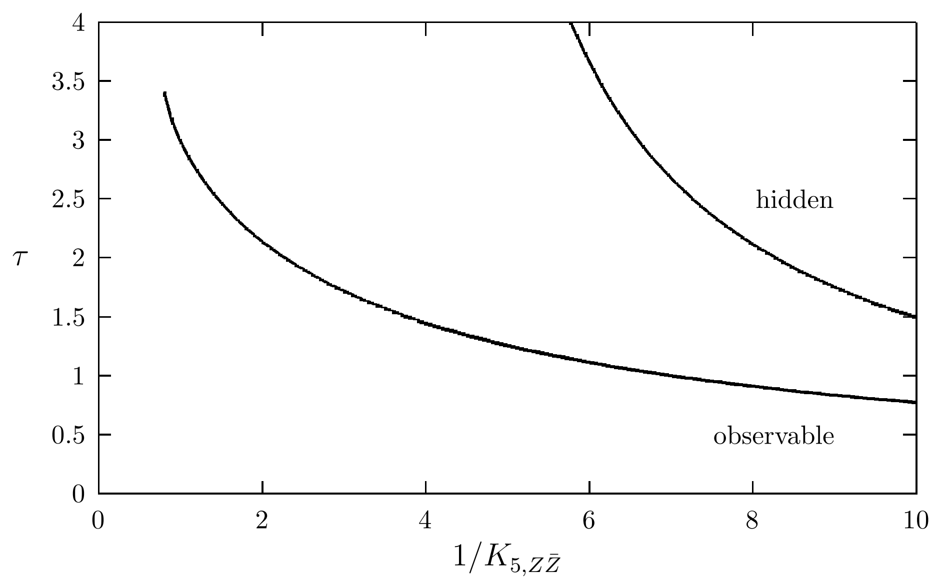

Fig.1: Solutions of $m_\Phi^2 = 0$ for $\cos\theta = 0$.

Similarly, we can investigate the condition $m_\Phi^2 = 0$ for other values of θ. Let us discuss the case with $\tan\theta = 1/\sqrt{3}$ as another example. We concentrate to the case that Φ belongs to the observable sector. When we fix $K_{5,Z\bar{Z}} = 1$, we have only the non-realistic solution, $(S+\bar{S})/(\alpha(T+\bar{T})) = 0$, i.e. $\alpha(T+\bar{T}) = 4$. Now let us vary $K_{5,Z\bar{Z}}$ fixing ϕ, e.g. $\tan\phi = 1$. Fig. 2 show the realistic solution against $K_{5,Z\bar{Z}}$ for $\tan\phi = 1$. In this case, the region $1/K_{5,Z\bar{Z}} > 5$ is favorable.

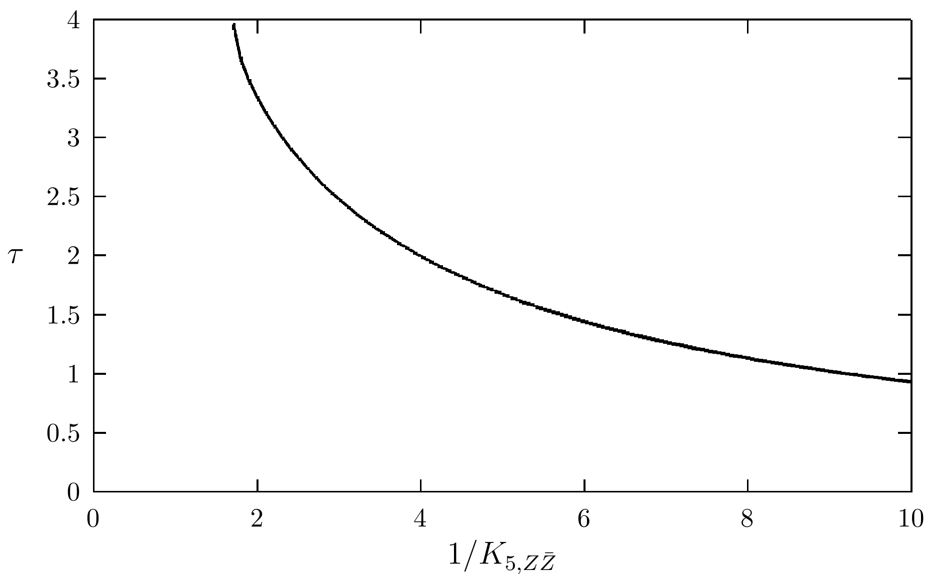

Fig.2: The solution $m_\Phi^2 = 0$ for $\tan\theta = 1$.

4 Conclusion

We have studied on the flatness condition within the framwework of M-theory with and without five-brane. In the case without five-brane, we can not obtain a realistic solution, because the condition (10) constrains severely. In the case with five-brane, the F-terms of five-brane moduli fields can also contribute the supersymmetry breaking and the vacuum energy. We have shown such effects are important to obtain the realistic solutions for $m_\Phi^2 = 0$. In particular, a large value of $1/K_{5,Z\bar{Z}}$ is favorable. In addition, the condition (10) is relaxed because of five-brane effects.

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